Code Design for MIMO Downlink with Imperfect CSIT

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***Abstract*—In this letter, we implement a simplified version of the Cover - van der Meulen - Hajek - Pursley (CMHP) coding originally characterized by Wajcer, Wiesel, and Shamai. The vector Gaussian broadcast channel with imperfect channel state information at the transmitter (CSIT) is considered where the transmitter only knows the channel mean and variance. Our focus is on the implementation and performance analysis of CMHP under the imperfect CSIT model using practical codes. Turbo codes described in IEEE 802.20 draft specification and quadrature amplitude modulation are used to implement CMHP. In order to find the optimal power allocation and beamforming vectors which maximize the sum rate with practical codes, we introduce the SINR penalty factor. The SNRs that achieve various target spectral efficiency are presented and analyzed.**

***Index Terms*—Imperfect channel state information, downlink, broadcast channel, superposition coding.**

I. INTRODUCTION

CONSIDER a multiuser communication scenario in which one transmitter with multiple antennas wishes to communicate with several single antenna receivers. The capacity of this multiuser communication channel also known as the vector Gaussian broadcast channel (GBC) has been shown to be achieved by Dirty Paper Coding (DPC) [1], [2]. Dirty Paper Coding, originally developed by Costa [3], is a coding technique which can precancel additive Gaussian interference perfectly over a Gaussian additive noise channel, where the interference is noncausally known at the transmitter but not at the receiver. Costa's DPC was implemented quite close to capacity by practical codes (see [4], [5] and references therein), however, the high complexity in implementing DPC motivates the investigation of more practical linear precoding schemes. Wajcer, Weisel, and Shamai [6] proposed an inner bound for the Cover-Van der Meulen-Hajek-Pursley (CMHP) [7]–[9] rate-region for the two user vector GBC, which utilizes common information that is decoded by both receivers via successive interference cancelation (SIC) decoders. The evaluation of CMHP and comparison among various schemes were done assuming perfect channel state information at the transmitter (CSIT), and CMHP was shown to outperform other schemes such as zero-forcing (ZF). Although assuming perfect CSIT is reasonable (and essential most of the time) for initial state of research, further investigation of *imperfect* CSIT (ICSIT) would be interesting since practical systems in general have imperfect channel state information due to estimation errors and feedback delay. Furthermore, the SIC structure of CMHP motivates the investigation of its performance under practical coding, especially under imperfect channel state information.

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In this letter, our focus is on the practical implementation of CMHP shown in [6]. Minimum mean square error (MMSE) beamforming (BF) and time division multiple access (TDMA), which are explained in Section III, are also implemented for comparison. Turbo codes described in IEEE 802.20 draft specification and quadrature amplitude modulation (QAM) are used to implement CMHP, MMSE beamforming, and TDMA under ICSIT. In order to find the optimal power allocation and BF vectors which maximize the sum rate for each transmission scheme with practical codes, we introduce the Signal-to-interference-and-noise ratio (SINR) penalty factor. For each scheme, we compare the corresponding SNRs that achieve a target spectral efficiency and the additional signal powers that are required to implement them are presented.

II. PRELIMINARIES

A. Notation

Vectors and matrices are expressed by boldface. The conjugate transpose of a is denoted by $a^\dagger$, and $E[\cdot]$ denotes the expectation of a random variable. The l_1 and l_2 norm are denoted by $|\cdot|$ and $\|\cdot\|$, respectively. We denote the convex hull operator and the trace of $\mathbf{A}$ by Co and $tr(\mathbf{A})$, respectively. Complex Gaussian and real Gaussian distributions with a mean of μ and variance of σ^2 are denoted by $\mathcal{N}_\mathcal{C}(\mu, \sigma^2)$ and $\mathcal{N}(\mu, \sigma^2)$, respectively. Notations $diag(\mathbf{x})$ and $\mathbf{x}^T$ are used to denote the diagonal matrix with elements from vector $\mathbf{x}$ and the transpose of $\mathbf{x}$, respectively.

B. Channel Model

We consider the vector Gaussian broadcast channel in which one transmitter with M antennas wishes to communicate with K single antenna receivers and is represented by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n},$$

where $\mathbf{y} \in \mathbb{C}^K$, $\mathbf{H} \in \mathbb{C}^{K\times M}$, $\mathbf{s} \in \mathbb{C}^M$, and $\mathbf{n} \in \mathbb{C}^K$. The vector $\mathbf{s}$ is the channel input with average power constraint $tr(E[\mathbf{s}\mathbf{s}^\dagger]) \leq P_{max}$ and $\mathbf{n}$ is the noise vector whose ith element $n_i \sim \mathcal{N}_\mathcal{C}(0, 1), i \in \{1, \ldots, K\}$ represents the additive Gaussian noise at receiver i. The row vector $\mathbf{h}_i$ of $\mathbf{H}$ is the channel vector between M transmitter antennas and the ith

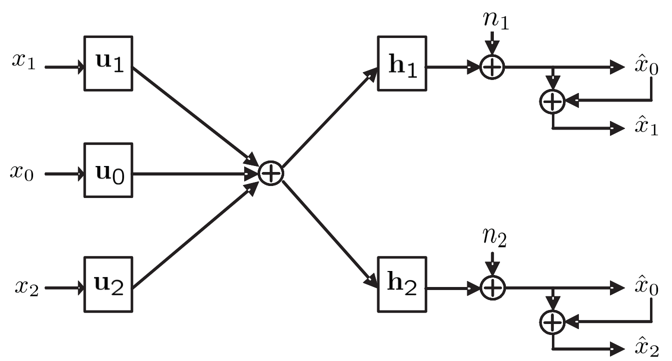

Fig. 1. CHMP for vector GBC.

receiver. Using the specified notations, the received signal at the ith user is expressed by

$$y_i = \mathbf{h}_i\mathbf{s} + n_i.$$

For the ICSIT channel model, the channel vector of receiver i is given by

$$\mathbf{h}_i = \hat{\mathbf{h}}_i + \tilde{\mathbf{h}}_i,$$

where $\hat{\mathbf{h}}_i$ is a deterministic vector and $\tilde{\mathbf{h}}_i$ is a circularly symmetric complex Gaussian random vector which follows the distribution $\mathcal{N}_{\mathcal{C}}(0, \sigma^2\mathbf{I})$. The vector $\hat{\mathbf{h}}_i$ models the channel estimate at the transmitter while $\tilde{\mathbf{h}}_i$ models the estimation error. We further assume that the receiver has access to both $\hat{\mathbf{h}}_i$ and $\tilde{\mathbf{h}}_i$, but the transmitter has access only to the distribution $\mathcal{N}_{\mathcal{C}}(\hat{\mathbf{h}}_i, \sigma^2\mathbf{I})$, $i \in \{1, \ldots, K\}$. Our ICSIT channel model is in between two extreme fading scenarios: Fast fading and slow fading. Ergodic fading and block fading which model extreme cases of fast and slow fading are represented by $\hat{\mathbf{h}}_i = 0, \sigma^2 = 1$ and $\mathbf{h}_i = \hat{\mathbf{h}}_i, \sigma^2 = 0$, respectively. Our model reflects realistic channels than the two extremes since practical communication systems usually experience fading between slow and fast.

III. Transmission Schemes

In this section Cover - van der Meulen - Hajek - Pursley (CMHP [7], [8], [9]) coding, MMSE, and TDMA transmission schemes are explained. CMHP with Gaussian signaling and SIC receivers was investigated in [6], and its system model for the 2 user vector GBC is shown in Fig. 1. The two user CHMP encoding makes use of three independently encoded data streams; two private streams which are intended to be decoded at each receiver and a common stream that is decoded by both receivers.

The data streams are encoded with a linear precoding structure and are superimposed for transmission. Formally, the input symbol of the jth stream takes the from of

$$\mathbf{s}_j = \mathbf{u}_j x_j, \quad j \in \{0, 1, 2\}$$

where $\mathbf{u}_j$ is the unit-norm beamforming vector for stream j, x_j is an information signal for stream j with $E[x_j^2] \le p_j$, and

$$\mathbf{s} = \sum_{j=0}^{2} \mathbf{s}_j.$$

The inputs x_1 and x_2 are private information sent to users 1 and 2, respectively. The 0th stream, x_0, which is called the common information, is the information that is *decoded* at both receivers. However, it is not necessarily the information that both users need, and can contain information for either user 1 or user 2 or both. Even if it has information for only user 1, user 2 benefits from canceling the interference that is part of user 1's stream through SIC. Let $\mathbf{p} = [p_0, p_1, p_2]^T$ be the power allocation vector which satify $|\mathbf{p}| \le P_{max}$, where an element p_j, $j \in \{0, 1, 2\}$ represents the power allocated to data stream j. For notational convenience let $\mathbf{U}$ denote the linear preprocessing matrix consisting of unit-norm column vectors $\mathbf{u}_j$, $j \in \{0, 1, 2\}$. The transmitted signal can be alternatively expressed by

$$\mathbf{s} = \mathbf{U}\mathbf{x},$$

where $\mathbf{x} = [x_0, x_1, x_2]^T$.

The collection of all rate triples (R_0, R_1, R_2) achieved by CMHP with Gaussian signaling under our ICSIT model can be represented as

$$\mathcal{R}^{cmhp} \triangleq Co \bigcup_{\substack{\mathbf{U},\mathbf{p} \\ \sum_{j=0}^{2} p_j \le P}} \left\{ (R_0, R_1, R_2) : \begin{array}{l} R_1 \le E\left[\log\left(1 + \alpha\frac{|\mathbf{h}_1\mathbf{u}_1|^2 p_1}{1+|\mathbf{h}_1\mathbf{u}_2|^2 p_2}\right)\right], \\ R_2 \le E\left[\log\left(1 + \alpha\frac{|\mathbf{h}_2\mathbf{u}_2|^2 p_2}{1+|\mathbf{h}_2\mathbf{u}_1|^2 p_1}\right)\right], \\ R_0 \le \min_{i=1,2} E\left[\log\left(1 + \alpha \mathrm{SINR}_i\right)\right] \end{array} \right\}. \tag{1}$$

where $\mathrm{SINR}_i \triangleq \frac{|\mathbf{h}_i\mathbf{u}_0|^2 p_0}{1+|\mathbf{h}_i\mathbf{u}_1|^2 p_1+|\mathbf{h}_i\mathbf{u}_2|^2 p_2}$, $\alpha = 1$, and R_i is the achievable rate for the ith stream. We will discuss the role of α in Section IV-B.

We will compare CMHP with two alternative transmission schemes. We define the MMSE scheme to be the same as CMHP, however, with p_0 set to zero. Thus, only private streams are utilized with linear precoding. The optimal beamforming solution with perfect CSIT is given by the MMSE beamforming vectors in the dual multiple access channel (MAC) domain [10]–[12]. Another baseline scheme we consider is time division multiple access (TDMA) which transmits one private stream at a time.

IV. Implementation of CMHP

In this section, we illustrate our main results. CMHP, MMSE, and TDMA are implemented using turbo codes and QAM modulation under ICSIT. We implement four points in the sum rate-SNR plane for each transmission scheme. The implementation results corresponding to the sum rate of $2, 4, 6, 8$ bps/Hz are analyzed. The organization of this section includes description of channel codes and the channel log-likelihood ratio (LLR) followed by an algorithm to find the optimal power allocation and BF vectors. We then show the numerical implementation results with analysis.

A. Channel Coding and Modulation

For implementation, the input signal is no longer Gaussian and is instead uniformly distributed in QAM constellation points. Moreover, a practical coding technique is used as a substitute to Gaussian codes which results in performance loss. We use the rate-$1/5$ turbo code described in the IEEE 802.20 draft specification where the turbo encoder consists of two systematic recursive convolutional encoders, and the generator matrix of each convolutional encoder is

$$\mathbf{G}(D) = \left[1 \quad \frac{1+D+D^3}{1+D^2+D^3} \quad \frac{1+D+D^2+D^3}{1+D^2+D^3}\right].$$

Periodic puncturing of parity bits is used to get the desired rate [13]. Non-punctured parity bits are scattered as much as possible within each parity branch. We occasionally use one parity branch per convolutional encoder to generate parity bits if it has better performance. Also, we note that the number of iterations in the decoder is 15. The SNR gap to capacity is about 2 dB when the bit error rate (BER) is 10^{-5} in the single input single output (SISO) AWGN channel.

The LLR of the turbo code is given by the following. The kth received symbol of the ith user is given by

$$y_{i,k} = \mathbf{h}_{i,k}\mathbf{u}_1 x_{1,k} + \mathbf{h}_{i,k}\mathbf{u}_2 x_{2,k} + \mathbf{h}_{i,k}\mathbf{u}_0 x_{0,k} + n_{i,k},$$

where $x_{j,k}$, $n_{i,k}$, and $\mathbf{h}_{i,k}$ are the symbol of stream j, Gaussian noise, and channel vector at time k, respectively. When decoding the common information, private signal components are treated as noise and for decoding private streams, the interfering private stream is treated as noise. The LLR of the common stream at decoder i is represented by

$$LLR_{0,i}(v_{0,k,l}) = \ln\left(\frac{\sum_{c\in S_0^+} \exp \frac{-|y_{i,k}-\mathbf{h}_{i,k}\mathbf{u}_0 c|^2}{|\mathbf{h}_{i,k}\mathbf{u}_1|^2 p_1 + |\mathbf{h}_{i,k}\mathbf{u}_2|^2 p_2 + 1}}{\sum_{c\in S_0^-} \exp \frac{-|y_{ik}-\mathbf{h}_{i,k}\mathbf{u}_0 c|^2}{|\mathbf{h}_{i,k}\mathbf{u}_1|^2 p_1 + |\mathbf{h}_{i,k}\mathbf{u}_2|^2 p_2 + 1}}\right)$$

and the LLR of private stream 1 is given by

$$LLR_1(v_{1,k,l}) = \ln\left(\frac{\sum_{c\in S_1^+} \exp \frac{-|y_1-\mathbf{h}_{1,k}\mathbf{u}_1 c|^2}{|\mathbf{h}_{1,k}\mathbf{u}_2|^2 p_2 + 1}}{\sum_{c\in S_1^-} \exp \frac{-|y_1-\mathbf{h}_{1,k}\mathbf{u}_1 c|^2}{|\mathbf{h}_{1,k}\mathbf{u}_2|^2 p_2 + 1}}\right),$$

where $v_{j,k,l}$ is the lth bit of the kth symbol of stream j. S_j^+ is a set of constellation points with $v_{j,k,l} = 1$ and S_j^- is the set of the rest of the constellation points. LLR_2 can be expressed in the same way as LLR_1 by exchanging subindices 1 with 2 in LLR_1. Since we assume perfect CSIR, the receiver calculates the LLR based on exact channel coefficients. We also note that, in the above expression, the interference is assumed to be Gaussian although it is uniformly distributed in the constellation points.

B. Resource Allocation

The optimal resources, e.g., BF vectors and power allocation that maximize the sum rate can be found by evaluating (1). However, since (1) is achieved by Gaussian codes, while evaluating optimal resources using (1), we would need to account for the suboptimality of the practical code. To this end, we introduce the SINR penalty factor denoted by α in (1) to account for this loss. We propose an algorithm for optimizing α in the following.

Initialization: Set the target sum rate, FER and α, β, g_{pre}.

1) Find resources such that the sum rate in (1) = target sum rate.
2) Implement a MIMO scheme using resources found in step 1.
3) $g = SNR_I - SNR_{\alpha=0\text{dB}}$
   if $g_{pre} < g$ then $\beta = -\beta/2$
4) $\alpha = \alpha + \beta$
   $g_{pre} = g$
5) Repeat steps 1 to 4 until g converges.

The parameter β is an increment on α, g_{pre} is some desired accuracy, and $SNR_{\alpha=0\text{dB}}$ is the lowest SNR achieving the target sum rate without penalty.

For some initial value of α, β, and g_{pre} the optimal resource allocation for a given SINR penalty is found by SINR balancing [6], [10] in the perfect CSIT case or by performing a grid search in the ICSIT case. Let SNR_I denote the SNR that achieves the target frame error rate (FER) when a transmission scheme is implemented with the given rate and resource allocation. Then we can calculate the SNR gap g between Gaussian codes and the turbo code for the given resource allocation, e.g.,

$$g = SNR_I - SNR_{\alpha=0\text{dB}}.$$

After adjusting α with the increment β, we repeat the steps until g converges to some desired value g_{pre}.

C. Numerical Evaluation

For numerical evaluation we set the channel parameters as

$$\hat{\mathbf{H}} = \begin{bmatrix} 1 & 0 \\ \cos(\pi/6) & \sin(\pi/6) \end{bmatrix}$$

and the covariance matrix of $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{h}}_2$ as $\text{diag}([0.008\ 0.008])$. The channel mean matrix is in between orthogonal and aligned channels. The estimation error variance σ^2 is 0.008 when the speed of a mobile user is about 80 km/h, the transmitter receives channel feedback every 0.25 ms, and the carrier frequency is 2.3 GHz. These parameters are based on WiBro standard specification [14].

The parameters used for implementation of CMHP, MMSE, and TDMA are shown in Table I, and we evaluate four implementation points corresponding to the target sum rates near $2, 4, 6, 8$ bps/Hz on the sum rate-SNR plane. The parameters are the spectral efficiency, code rate, modulation, frame length, and α of each stream. To determine these parameters, we use the algorithm described in IV-B. We set the frame length to be around 4000 for all schemes for fair comparison. FER curves for each stream of the implemented points are plotted in Fig. 2. Also, we note that the FER for the common stream is the left curve among the two FERs that correspond to the CMHP evaluations in Fig. 2. The SNR gap between the implementation result and evaluation assuming Gaussian signaling at the same sum rate are represented in Table II.

Implementation results with sum rates around 8 bps/Hz are represented by C_1, M_1, and T_1 for CMHP, MMSE, and TDMA, respectively in Table II. We note that M_1 does not fall back to TDMA despite the fact that its α is large in this regime. Since the gap of M_1 is larger than that of C_1, the SNR gap between these two transmission schemes become larger than the gap when we evaluate assuming Gaussian signaling by 1.14 dB. Thus, we can see that the relative performance

TABLE I
PARAMETERS FOR IMPLEMENTATION

ICSIT		R_T	R_I	γ	Mod	FL	α [dB]
C_1	$R_1(R_2)$	2.70	2.636	29/44	16QAM	4006	-1.87
	R_0	2.71	2.823	12/17	16QAM		
M_1	$R_1(R_2)$	4.076	4.08	17/25	64QAM	4001	-2.76
T_1	R_1	8.00	8.00	8/10	1024QAM	4000	-2.77
C_2	$R_1(R_2)$	1.78	1.75	3/7	16QAM	4001	-1.7
	R_0	2.42	2.40	3/5	16QAM		
$M_2(T_2)$	$R_1(R_2)$	6.00	6.00	3/4	256QAM	4001	-2.62
C_3	$R_1(R_2)$	0.85	0.9	9/20	4QAM	4001	-1.84
	R_0	2.29	2.33	7/12	16QAM		
$M_3(T_3)$	R_1	4.00	4.00	2/3	64QAM	4001	-2.20
$C_4(M_4, T_4)$	R_1	2.00	2.00	1/2	16QAM	4000	-1.84

R_T is the target rate of each stream, and R_I is the implemented rate of each stream in [bps/Hz]. γ is the coding rate, Mod is the modulation scheme, α is the SINR penalty, and 'FL' is the frame length. Notations C, M, and T represent CMHP, MMSE, and TDMA, respectively.

TABLE II
IMPLEMENTATION RESULTS

ICSIT	Sum rate [bps/Hz]	SNR$_I[dB]$	SNR$_E[dB]$	Gap
C_1	8.09	22.74	19.60	3.14
M_1	8.16	27.10	22.80	4.30
T_1	8.00	26.90	24.06	2.84
C_2	5.83	17.94	15.05	2.89
M_2	6.00	20.72	17.48	3.24
T_2	6.00	20.72	17.98	2.74
C_3	4.13	13.48	11.08	2.40
$M_3(T_3)$	4.00	13.98	11.72	2.26
$C_4(M_4, T_4)$	2.00	6.61	4.72	1.89

SNR$_I$ and SNR$_E$ are the corresponding SNRs achieving the sum rate using the turbo code and Gaussian codes, respectively. The gap is defined as SNR$_I$ $-$ SNR$_E$.

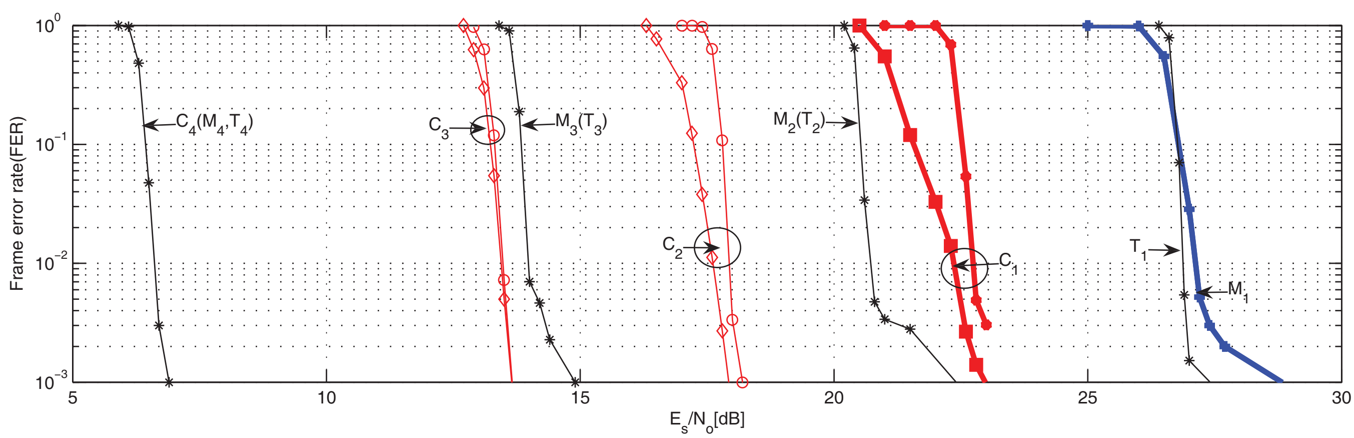

Fig. 2. FER curve for each stream of CMHP, MMSE, and TDMA.

gain of CMHP compared with MMSE in the case where practical implementation is considered is large in the high SNR regime. In addition, we observe that the performance of MMSE degrades seriously when it is implemented assuming that the transmitter has imperfect channel information. The gap of T_1 is the smallest among the three, but CMHP is still better in the sense of required SNR.

For 6 bps/Hz and below, the implementation results of MMSE and TDMA are the same since MMSE falls back to TDMA even in the regime where MMSE is better than TDMA assuming Gaussian signaling. Notations C_2, M_2, and T_2 denote the points corresponding to the sum rate near 6 bps/Hz for CMHP, MMSE, and TDMA, respectively. When we implement CMHP and MMSE around 6 bps/Hz, the SNR gap between the two increases by 0.33 dB compared to the SNR gap evaluated using Gaussian codes.

T_3 is the implementation result for MMSE and TDMA and C_3 is the implementation of CMHP with sum rate near 4 bps/Hz. In this case, the SNR gap between CMHP and the others shrink by about 0.2 bps/Hz when they are implemented, but CMHP still has the best performance. The performance of all three transmission schemes fall back to TDMA in the low SNR regime.

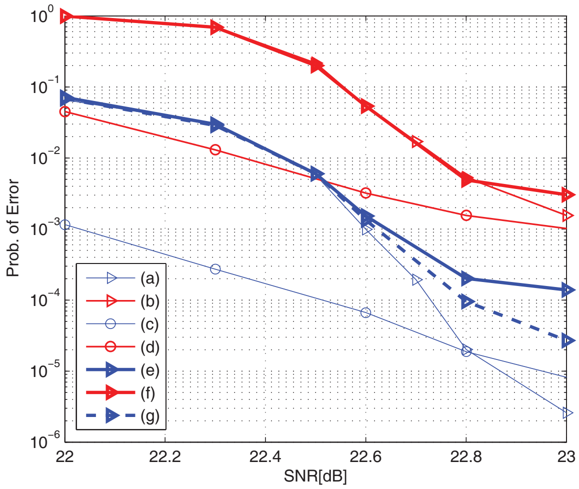

Fig. 3. FER curves for C_1 with various decoding assumptions: (a) and (b) are the BER and FER for private streams assuming perfect SIC, respectively. (c) and (d) are the BER and FER for the common information, respectively. (e) and (f) are the BER and FER for private streams with re-encoding, respectively. (g) and (f) are the BER and FER for private streams without re-encoding, respectively.

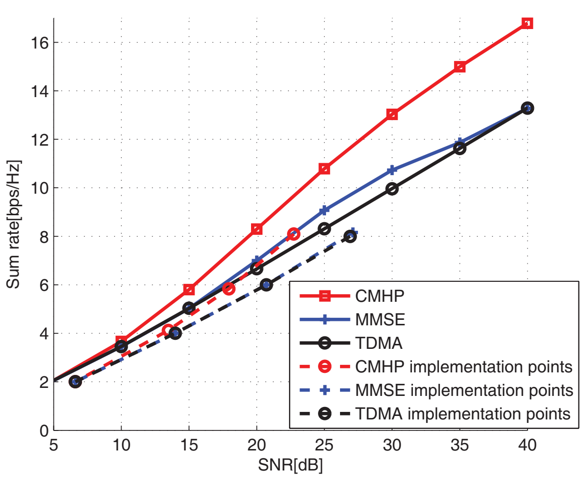

Fig. 4. Solid lines are the evaluations assuming Gaussian codes and the dotted lines are the implementation results using turbo codes.

D. CMHP FER

First, let us focus on the error propagation in SIC. Since each receiver performs SIC, decoding error resulting from the common stream will be propagated to those of the private streams. In other words, if there is a frame error in the common stream, it increases the frame error rate for the private streams. Thus, FER for the private stream of each user will be approximately equal to the sum of the FER of its private stream assuming perfect SIC and the FER of the common stream. However, BER is a different story. If the decoded common stream has some bit errors, the re-encoded output codeword may differ from the correct codeword due to the feedback structure of the encoder. As a result, BER for private streams may be very high. To avoid serious error propagation caused by the re-encoding process, we instead subtract the estimated *codeword* directly without re-encoding. The target FER for the common stream is set to 10^{-2}. Fig. 3 shows the severity of error propagation for each case. We can see that SIC without re-encoding has better performance. The FER of the private stream is also reduced for the same reason, but the effect of SIC without re-encoding is not shown in this figure since the FER of the private streams assuming perfect SIC is not much lower than that of the common stream.

Another observation is that the slopes of the FER curves of the common and private streams are different in the high SNR regime as shown in Fig. 2. The reason for a steeper slope for the private stream than that of the common stream is because private streams are less affected by interference. As shown in (1), without interference from the common stream, only the private stream of one user interferes with the other user when private stream is decoded and also the interference power is not high due to the BF vectors being close to that of ZF. The lower the interference, the more SINR is varied as SNR changes. Therefore, as SNR increases, SINR for private streams increase more than that of the common stream, and so FER decrease faster. However, the slopes of the FER curves for private and common streams approach a similar level as SNR decreases since SINR get closer to SNR.

V. Conclusion

The performance of the achievable scheme proposed by Wajcer, Wiesel, and Shamai has been investigated under practical coding. For optimal resource allocation we proposed the SNR penalty factor to account for the sub-optimality of practical codes. Numerical simulations were performed for the 2×1 two user vector GBC under ICSIT. Simulation with parameters based on the 802.20 Wibro specifications showed that by utilizing an addition common stream, a significant gain was attained over MMSE and TDMA in the high SNR regime.

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